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The Effect of Bank Angle and Weight on the Minimum Control Speed V_{MCA} of an Engine-out Airplane for preparing experimental flight-tests to determine V_{MCA}

References

1. Report: *Airplane Control after Engine Failure*. AvioConsult, June 2005, www.avioconsult.com.
2. Paper: *Staying Alive With a Dead Engine*. AvioConsult, February 2006.
3. *Flying Qualities Testing, Failure State Testing, Chapter 32*. USAF Test Pilot School, Edwards Air Force Base, CA, USA.
4. *Flight Characteristics I, Part B* by Prof. Dr. Ir. O.H. Gerlach, Technical University Delft, NL.
5. Airplane Design; *Stability and Control During Steady Straight Flight*, Chapter 4, Dr. Jan Roskam, DAR Corporation, Kansas.

Introduction

In the report *Airplane Control after Engine Failure* (ref. 1) and in the Paper *Staying Alive with a Dead Engine* (ref. 2), the effect of bank angle and weight on V_{MCA} (and V_2) was presented in figures. In this paper, the analysis to derive the figures is presented.

1. The relation between airspeed, bank angle and weight

1.1. In ref's 3 to 5, equations are presented for equilibrium flight. These equations, included below, can be used to calculate and plot sideslip angle β , aileron deflection δ_a and rudder deflection δ_r of a multi-engine airplane at varying weights W , airspeeds V and thrust levels N_T (read altitudes) while the cg is aft (as used to determine V_{MCA}), one engine is inoperative and the remaining engines are developing full thrust. The equations describe unaccelerated, 1g, constant heading flight.

The results of the analysis are normally used before commencing V_{MCA} flight testing to predict the lowest calibrated airspeed at which the airplane can be trimmed (= controlled) while an engine is inoperative and to be aware of the expected limitations. Refer to ref's 3 to 5 for a detailed analysis. In the equations, dynamic pressure (q), weight ($m \cdot g$), thrust (N_T), and bank angle (ϕ) are used as variables. The equations are for a turbojet airplane, not for a propeller airplane where thrust augments the lift (propulsive lift). Trim as used here means the trim setting plus the manual control inputs required to maintain the one engine inoperative equilibrium flight.

1.2. The equations out of ref's 3 to 5 are presented here. In deriving these equations, the small angle assumption is used; $\cos \theta = 1$ and $\sin \phi = \phi$ (in radians) because θ is usually less than 15 degrees and ϕ is less than 5 degrees.

$$C_{y\beta} \beta_{trim} + C_{y\delta_a} \delta_{a_{trim}} + C_{y\delta_r} \delta_{r_{trim}} = \frac{-F_y}{\bar{q}S} - \frac{mg\Phi}{\bar{q}S} - C_{y_0}$$

$$C_{l\beta} \beta_{trim} + C_{l\delta_a} \delta_{a_{trim}} + C_{l\delta_r} \delta_{r_{trim}} = \frac{-L_T}{\bar{q}Sb} - C_{l_0}$$

$$C_{n\beta} \beta_{trim} + C_{n\delta_a} \delta_{a_{trim}} + C_{n\delta_r} \delta_{r_{trim}} = \frac{-N_T}{\bar{q}Sb} - C_{n_0}$$

1.3. Because there are four variables in these three equations (ϕ , β , δ_a and δ_r), many states of equilibrium are possible, but only the cases in which $\phi = 0^\circ$ and $\beta = 0^\circ$ are of most interest. Zero bank angle is easy to fly (IMC) and zero sideslip causes the total drag of the airplane to be minimum, which is favorable to the remaining climb performance after engine failure.

1.4. These simultaneous linear equations can be solved for angle of sideslip (β), aileron deflection (δ_a) and rudder deflection (δ_r) using Cramers rule. The resulting equations, presented below, define β , δ_a and δ_r required to trim an airplane whether or not an engine is inoperative. Trim here means the total control deflection for maintaining equilibrium (balance of forces and moments).

$$\Delta = \begin{vmatrix} C_{l\beta} & C_{l\delta_a} & C_{l\delta_r} \\ C_{n\beta} & C_{n\delta_a} & C_{n\delta_r} \\ C_{y\beta} & C_{y\delta_a} & C_{y\delta_r} \end{vmatrix}$$

$$\beta_{trim} = \frac{\begin{vmatrix} \frac{-L_T}{\bar{q}Sb} - C_{l_0} & C_{l\delta_a} & C_{l\delta_r} \\ \frac{-N_T}{\bar{q}Sb} - C_{n_0} & C_{n\delta_a} & C_{n\delta_r} \\ \left(\frac{-F_y}{\bar{q}S} - \frac{mg\Phi}{\bar{q}S} - C_{y_0}\right) & C_{y\delta_a} & C_{y\delta_r} \end{vmatrix}}{\Delta} \text{ radians}$$

$$\delta_{r_{trim}} = \frac{\begin{vmatrix} C_{l\beta} & C_{l\delta_a} & \frac{-L_T}{\bar{q}Sb} - C_{l_0} \\ C_{n\beta} & C_{n\delta_a} & \frac{-N_T}{\bar{q}Sb} - C_{n_0} \\ C_{y\beta} & C_{y\delta_a} & \left(\frac{-F_y}{\bar{q}S} - \frac{mg\Phi}{\bar{q}S} - C_{y_0}\right) \end{vmatrix}}{\Delta} \text{ radians}$$

$$\delta_{a_{trim}} = \frac{\begin{vmatrix} C_{l\beta} & \frac{-L_T}{\bar{q}Sb} - C_{l_0} & C_{l\delta_r} \\ C_{n\beta} & \frac{-N_T}{\bar{q}Sb} - C_{n_0} & C_{n\delta_r} \\ C_{y\beta} & \left(\frac{-F_y}{\bar{q}S} - \frac{mg\Phi}{\bar{q}S} - C_{y_0}\right) & C_{y\delta_r} \end{vmatrix}}{\Delta} \text{ radians}$$

1.5. These equations can be used to prepare for flight testing V_{MCA} to predict the lowest calibrated airspeed at which the airplane can be controlled while an engine is inoperative, to identify the limitations that might occur during flight testing. In the examples that are presented below, the following stability derivatives of a 4-engine turbojet airplane (B707, DC-8 class), determined at an aft cg location and in the approach configuration, were used:

$C_{l\beta} =$	-0.0849	per radian	$C_{n\beta} =$	0.1920	per radian	$C_{y\beta} =$	-0.8660	per radian
$C_{l\delta_a} =$	0.0856	per radian	$C_{n\delta_a} =$	0.0106	per radian	$C_{y\delta_a} =$	0.0000	per radian
$C_{l\delta_r} =$	0.0218	per radian	$C_{n\delta_r} =$	-0.1660	per radian	$C_{y\delta_r} =$	0.3700	per radian
$C_{l_0} =$	0.0000	per radian	$C_{n_0} =$	0.0000	per radian	$C_{y_0} =$	0.0000	per radian

1.6. After substituting these stability derivative data in the equations that are presented in paragraph 1.4, the following equations are obtained for β , δ_a and δ_r :

(W in lb, ϕ in radians, ρ_0 in slug/ft³, V in ft/sec)

$$\beta = (0.052300 \cdot W \cdot \phi + 0.000808 \cdot N_T) / \frac{1}{2} \rho_0 V^2$$

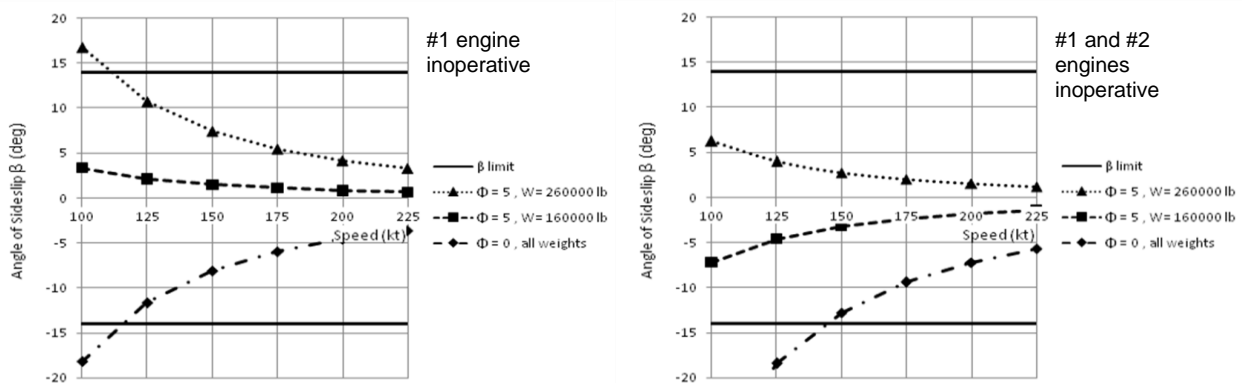
$$\delta_a = (0.035900 \cdot W \cdot \phi + 0.000319 \cdot N_T) / \frac{1}{2} \rho_0 V^2$$

$$\delta_r = (0.063026 \cdot W \cdot \phi + 0.001890 \cdot N_T) / \frac{1}{2} \rho_0 V^2$$

1.7. N_T for this 4-engine airplane at sea level is (l is moment arm):

- $N_T = l_x \cdot F_y - l_y \cdot F_x = 0 - (-45 \text{ ft}) \cdot (-17,000 \text{ lb}) = -765,000 \text{ ft-lb}$ for #1 engine inoperative;
- $N_T = -1,207,000 \text{ ft-lb}$ for both #1 and #2 inoperative (for calculating V_{MCA2}).

1.8. As an example, the angle of sideslip β was calculated for different speeds, bank angles, and weights, at sea level. The plots presented below, show the sideslip angle versus airspeed for a B707, DC-8 class airplane for sea level, high and low weights and two bank angles, wings level ($\phi = 0$) and $\phi = 5$ degrees away from the failed engine. Similar plots are also made for δ_a and δ_r . These plots are made before beginning V_{MCA} flight testing to be aware of the expected sideslip and control limits and speeds. The left-hand plot is for one engine inoperative (# 1), the right-hand plot for two engines (#1 and #2) inoperative. ($+\beta =$ wind right ear; $+\phi =$ to right)



2. The effect of bank angle and weight on V_{MCA}

2.1. In accordance with the definition of V_{MCA} in FAR/ CS 23 and 25, the actual (lowest) airspeed, at which one of the control surface maxima aileron (δ_a) or rudder (δ_r) is reached, is V_{MCA} . In addition, the civil regulations allow a maximum rudder force of 150 lb (667 N) and a maximum roll control force of 25 lb (112 N).

Military specifications require that roll control shall not exceed either this maximum roll control force limit or 75% of the available control power, leaving room for countering gusts, etc.; the maximum pedal force is 180 lb.

Furthermore, the vertical fin should not stall, so there is a maximum angle of sideslip β to take into account as well during predicting V_{MCA} .

2.2. The pedal and wheel force limits were not included in the equations, but should be subject of flight-testing; the V_{MCA} determined in-flight may therefore be higher than the calculated or predicted V_{MCA} . There must remain a reason for performing actual V_{MCA} flight testing. For the pur-

pose of the report (ref. 1) though, this analysis, even without the availability of control force data, provides already very interesting data that can be used to show the effect of bank angle and weight on V_{MCA} .

2.3. The equations presented above cannot only be used to calculate β , δ_a and δ_r from thrust, weight, bank angle and airspeed, but can also be reworked to calculate the calibrated airspeed for the given maximum (mechanical) deflections of rudder and aileron and the maximum allowable sideslip angle β ('vertical' angle of attack) before the vertical fin stalls. This way, the speeds for which β , δ_a and δ_r reach their maximum authorized or available values can be predicted.

2.4. The equations of paragraph 1.6 can be rearranged to be solved for the speed at which either the maximum deflection limit of rudder or ailerons is reached, or the maximum angle of sideslip is reached. For the sample airplane, these limits are: β is max. ± 14 degrees, δ_a is max. ± 20 degrees and δ_r is max. ± 30 degrees. Weight W should be entered in lb; ϕ , $\delta_{a \max}$, $\delta_{r \max}$ and β in radians; $\rho_0 = 0.0023769$ slug/ft³. The derived equations are:

- $V_{\max. \beta} \text{ (kt)} = (1/1.689) \cdot (\sqrt{((0.052300 \cdot W \cdot \phi + 0.000808 \cdot N_T) / (\beta_{\max} \cdot 0.5 \rho_0))})$
- $V_{\max. \delta_a} \text{ (kt)} = (1/1.689) \cdot (\sqrt{((0.035900 \cdot W \cdot \phi + 0.000319 \cdot N_T) / (\delta_{a \max} \cdot 0.5 \rho_0))})$
- $V_{\max. \delta_r} \text{ (kt)} = (1/1.689) \cdot (\sqrt{((0.063026 \cdot W \cdot \phi + 0.001890 \cdot N_T) / (\delta_{r \max} \cdot 0.5 \rho_0))})$

For deriving these equations, the small bank angle assumption ($\sin \phi = \phi$ (in radians)) was used: ϕ should be less than 5 degrees. However, the difference between $\sin \phi$ and ϕ (in radians) when $\phi = 15$ degrees is only 0.00298. For the purpose of only illustrating the effect of bank angle on V_{MCA} it is considered acceptable to show data for bank angles up to 15 degrees.

2.5. The actual V_{MCA} , while bank angle and/ or weight vary and the thrust N_T is maximum, is the highest speed of either one of the speeds calculated using the three equations above, because then one of the FAR/ CS control limits is reached, or the angle of sideslip β is on the limit. The airspeed should not be decreased any further than this speed, or the airplane will be out of control because there is either no rudder or aileron control power left, or the increased angle of sideslip causes the vertical fin to stall. As mentioned above, rudder and aileron control forces are not included in the calculation, which might lead to a slightly higher V_{MCA} during flight testing.

2.6. The equations presented above can be entered in a spreadsheet that allows for solving for airspeed V and plotting diagrams showing the effect of bank angle into or away from the inoperative engine(s) on actual V_{MCA} for several weights. Also a diagram can be plotted that shows the relation of V_{MCA} versus weights for several bank angles. A few calculated figures for both one and two engines inoperative of the sample airplane are presented on the next page.

3. Conclusion

3.1. The results of the analysis show that bank angle and weight obviously have great effect on the actual V_{MCA} of a multi-engine airplane. It proves that the V_{MCA} that is presented in the Airplane Flight Manual is valid only if the same bank angle is being maintained that was used to determine V_{MCA} , i.e. during unaccelerated, 1g, constant heading flight and definitely not during turns. Any other bank angle will result in a higher actual V_{MCA} and might lead to control problems if the power setting is high and the airspeed and weight are as low as used during V_{MCA} testing.

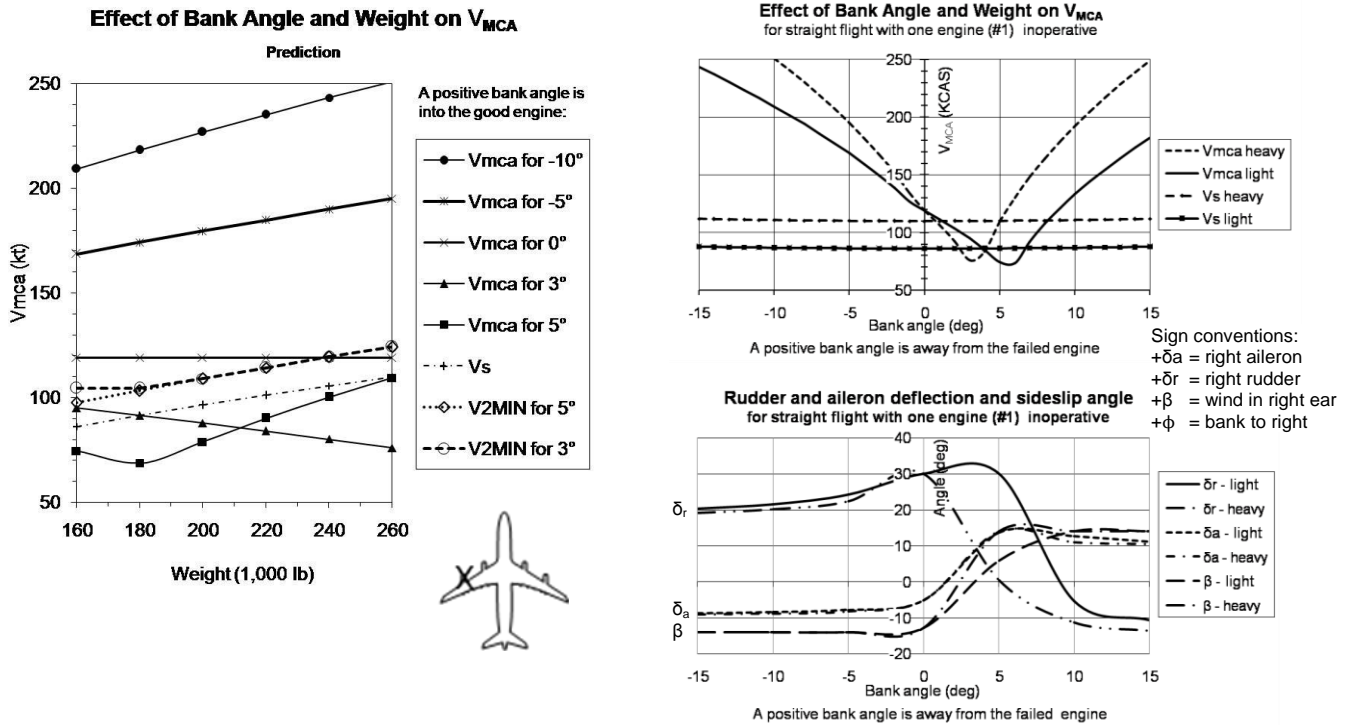
3.2. This analysis should be used to improve engine emergency procedures and could be used as well to explain the real cause of many accidents after engine failure.

3.3. Refer to ref. 1 for additional information on V_{MCA} and explanation of the presented figures and data.

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The figures presented below are calculated using the analysis presented in this paper. The figures are valid for unaccelerated, 1g, constant heading flight. The small angle assumption presented in paragraph 2.4 applies.

1. Effect of weight and bank angle on V_{MCA} , one engine (#1) inoperative:



2. Effect of weight and bank angle on V_{MCA} , two engines (#1 and #2) inoperative:

